OPTIMIZING THE ELECTRICAL SUBSTITUTION PROCEDURE FOR A CAVITY RADIOMETER HAVING RADIATIVE AND CONVECTIVE—RADIATIVE HEAT TRANSFER

V. I. Polevoi, L. A. Nazarenko, and V. M. Guzei

UDC 535.214.4

A spherical cavity is used to consider optimizing a replacement equivalence criterion, which enables one to minimize the systematic error in such a radiometer.

A cavity radiometer employing electrical substitution is commonly employed as a standard or substandard means of measuring thermal and laser radiation fluxes [1]. This imposes stringent requirements on the accuracy, which is governed particularly by the systematic error arising from the nonequivalent temperature patterns in the detector due to the radiation flux and the replacement electrical power.

Studies have been made on that error [2, 3] for a spherical cavity whose heating was recorded with a single thermocouple and with convective—radiative heat transfer, or in [4] for radiative heat transfer for a cavity in a vacuum chamber. The basic systematic error can be minimized by the best choice of the thermocouple coordinate, which can be based on preliminary information on the aperture angle for the input radiation, the relative distribution of the cavity wall illumination E(x), and the mode of radiation flux scattering within the receiving element.

If such information is lacking or very inaccurate, it is better to use a set of thermocouples which enable one to integrate the temperature pattern of the outer surface no matter what the radiation flux distribution. The thermocouples are usually connected in series as a thermobattery in a spherical or other type of cavity radiometer, and equality of the readings on irradiation and on replacement is taken as a criterion for equivalence in the electrical replacement procedure.

With a spherical shell and a temperature pattern symmetrical about the axis, the thermocouples may be located in a plane passing through the symmetry axis on the outer surface of the detecting element at points whose coordinates θ_k correspond to Δx_k = const, where $x = \cos\theta$, $k = 1, 2, \ldots, N$. That thermocouple positioning provides equal areas for the annular zones ΔS_k on the surface per thermocouple. With Δx_k = const, we have that

$$\Delta S_k = 2\pi R^2 (\cos\theta_k - \cos\theta_{k+1}) = 2\pi R^2 \Delta x_k = \text{const},$$

in which $\Delta x_k = x_k - x_{K+1}$, where $x_1 = 1$, since the polar angle θ in the spherical system is here reckoned from the ray passing from the center of the sphere through the point lying opposite the center in the entrance aperture.

It is shown below that only with that thermocouple disposition can equality in the thermobattery readings be considered as a correct criterion for replacement equivalence (for convective—radiative heat transfer closely similar to linear). That is the basis for choosing this design.

However, in general, the detector positioning and the reading processing may have substantial effects on the basis systematic error, so a thermobattery used with arithmetic summation, which imposes constraints on the disposition, cannot be considered as the only arrangement and particularly not the optimal one of realizing the replacement in the sense of minimizing that error.

It is not a rational procedure to reduce the error by increasing the total number N of thermocouples because this increases various errors, the main one of which is the error caused by the lack of equivalence between the conductive heat transfer in the thermobattery block on irradiation as opposed to substitution, together with the error arising in measure-

Metrologiya Cooperative, Kharkov. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 60, No. 2, pp. 310-317, February, 1991. Original article submitted January 18, 1990.

ments in air from the unstable local heat fluxes around the thermocouples. Also, increasing N complicates the design and raises the probability of differences in thermal resistance between the couples, which also reduces the accuracy.

Similar problems arise for other cavity shapes, so reducing the basic systematic error without increasing the total number of couples is a common problem.

For a spherical cavity with a small hole ($d^2 \ll 4R^2$) and convective—radiative transfer in a thermostatically controlled air or liquid medium, the conditions may allow linearization in accordance with Newton's law, and then the power dissipated by the outer surface of the receiving element in the steady state is

$$P = 2\pi R^2 \alpha \int_{-1}^{1} [T(R,\ x) - T_0]\, dx, \tag{1}$$

in which the effective heat-transfer coefficient α may include convective and radiative components [2].

For radiative heat transfer in a vacuum chamber having temperature T_0, correspondingly

$$P = 2\pi R^2 \varepsilon_2 \sigma \int_{-1}^{1} [T^4(R,\ x) - T_0^4]\, dx. \tag{2}$$

We transfer in (1) to numerical integration by the rectangle method with nodes uniformly arranged along x and use the fact that the heater provides almost uniform heating for the detecting element in replacement mode to get on equating the heat-transfer power levels on irradiation and substitution that

$$\sum_{k=1}^{N} V_1(P_1,\ x_k) = NV_2(P_2), \tag{3}$$

in which

$$V_1(P_1,\ x_k) = \beta [T_1(P_1,\ R,\ x_k) - T_0];\quad V_2 = \beta [T_2(P_2,\ R) - T_0]. \tag{4}$$

Then (3) corresponds to equal readings from the thermobattery consisting of N thermocouples in series, from which one gets equivalence between the radiative and electrical power levels. As N is always limited, the radiation power P_1 and the replacement power P_2 meeting (3) for given N and $\theta_k = \arccos x_k$ may differ considerably, which is the reason for the systematic error defined by

$$\mu_0 = \frac{P_1 - P_2}{P_2} = 1 - \frac{P_2}{P_1}. \tag{5}$$

With linear heat transfer, the relative temperature distribution over the outer surface in a multilayer spherical shell is [5]

$$U_1(1,\ x) = \frac{T_1(P_1,\ R,\ x)}{T_0} = 1 + \frac{P_1}{4\pi\lambda_s R T_0 \mathrm{Bi}} + \frac{\varepsilon_1 P_1}{4\pi\lambda_s T_0 R} \sum_{m=1}^{\infty} C_m(x), \tag{6}$$

in which

$$C_m(x) = \frac{(2m+1)^2 R_{01}^m F_m P_m(x)}{L_m(m+1)(m+\mathrm{Bi}) - N_m m(m+1-\mathrm{Bi}) R_{01}^{2m+1}};$$

$$F_m = \frac{\int_{x_0}^{1} E(x) P_m(x)\, dx}{\int_{x_0}^{1} E(x)\, dx};$$

$R_{01} = R_1/R$; $\mathrm{Bi} = \alpha R/\lambda_s$ is the Biot number, and L_m and N_m are coefficients dependent on the number of layers and the thicknesses and thermal conductivities, which have been given for a two-layer model in [3], and for a three-layer one in [5], while for a single-layer one $L_m = N_m = 1$, $x_0 = \cos\theta_0$.

The relative temperature of the outer surface with uniform heating by the substitution power P_2 is [5]

$$U_2(1) = \frac{T_2(P_2,\ R)}{T_0} = 1 + \frac{P_2}{4\pi R\lambda_s T_0 \mathrm{Bi}}. \tag{7}$$

We substitute for $T_2(P_2, R)$ from (3) into (7) and determine P_2 from (7) and substitute into (5) to get

$$\mu_{01}^{x} = 1 + \frac{4\pi R T_0 \lambda_s \mathrm{Bi}}{NP_1}\left[N - \sum_{k=1}^{N} U_1(1,\ x_k)\right]. \tag{8}$$

We then substitute $U_1(1, x)$ for $x = x_k$ from (6) into (8) to get finally

$$\mu_{01}^{x} = -\frac{\varepsilon_1 \mathrm{Bi}}{N}\sum_{k=1}^{N}\sum_{m=1}^{\infty} C_m(x_k). \tag{9}$$

The subscript 1 in μ_{01}^{x} emphasizes that (9) is applicable only in the linear heat-transfer approximation, while the superscript x indicates that the thermocouples are uniformly distributed in x, i.e., $\Delta x_k = 2/N$, $x_k = -1 + k\,2/N$. With any other thermocouple positioning, equal readings from the thermobattery in the two states cannot serve as criterion for equivalence in the replacement because (3) corresponds to approximate equality in the heat outputs only for $\Delta x_k = \mathrm{const}$. If, for example, the thermocouples are placed with a uniform step in θ, i.e., $\theta_k = \pi - k\,\pi/N$, while $\Delta\theta_k = \pi/N$, the equation becomes

$$\sum_{k=1}^{N} V_1(P_1,\ \theta_k)\sin\theta_k\Delta\theta_k = V_2(P_2)\sum_{k=1}^{N}\sin\theta_k\Delta\theta_k. \tag{10}$$

As $\Delta\theta_k$ is constant, we convert to the variable x to get

$$\sum_{k=1}^{N} V_1(P_1,\ x_k)\sqrt{1-x_k^2} = V_2(P_2)\sum_{k=1}^{N}\sqrt{1-x_k^2}. \tag{11}$$

Then the substitution procedure should be based on implementing (11) instead of (3), while the basic systematic error μ_{01}^{θ} is correspondingly

$$\mu_{01}^{\theta} = -\frac{\mathrm{Bi}\sum_{k=1}^{N}\sum_{m=1}^{\infty} C_m(x_k)\sqrt{1-x_k^2}}{\sum_{k=1}^{N}\sqrt{1-x_k^2}}. \tag{12}$$

With radiative (nonlinear) heat transfer, equal thermobattery readings even with $\Delta x_k = \mathrm{const}$ do not imply a criterion for equivalence in the replacement because the power passing from the surface of the sphere to the vacuum chamber cannot be expressed in terms of the sum of the emf readings in (4), which are linearly dependent on temperature. The corresponding criterion should be based on (2) and for $\Delta x_k = \mathrm{const}$ is formulated as

$$\sum_{k=1}^{N}\left[\frac{V_1(P_1,\ x_k)}{\beta} + T_0\right]^4 = N\left[\frac{V_2(P_2)}{\beta} + T_0\right]^4. \tag{13}$$

The basic systematic error μ_{02}^{x} is then

$$\mu_{02}^{x} = 1 + \frac{4\pi R^2 \varepsilon_2 \sigma T_0^4}{NP_1}\left[N - \sum_{k=1}^{N} U_{1p}(1,\ x_k)\right]. \tag{14}$$

The subscript 2 in μ_{02}^{x} indicates radiative heat transfer.

It is not possible to write the μ_{02}^{x} in analytic form as in (9) because the corresponding nonlinear boundary-value problem can be solved only numerically [4]. However, one can substitute the numerical values for $U_{1p}(1, x)$ for $x = x_k$ into (14) from the solution derived in [4] to examine the dependence of μ_{02}^{x} on N and θ_k.

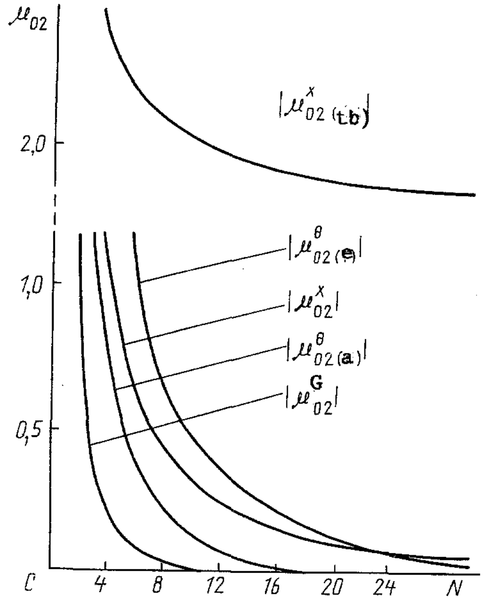

Fig. 1. The N dependence of $|\mu_{02}|$ for radiative transfer, μ_{02} in %.

If a thermobattery is used here, i.e., (3), there would be an error $\mu_{02(tb)}^x$, defined by

$$\mu^x_{02(tb)} = 1 + \frac{4\pi R^2 \varepsilon_2 \sigma T_0^4}{N^4 P_1}\left\{N^4 - \left[\sum_{k=1}^{N} U_{1p}(1,\ x_k)\right]^4\right\}. \tag{15}$$

Calculations from (14) and (15) for a model whose parameters are given in [4] show that realizing (13) instead of (3) with p = 10 W, θ_0 = 10°, and E(x) ≈ const leads for example for N = 8 to almost a fivefold reduction in the basic systematic error. The relative advantage in the accuracy increases further with N. Figure 1 shows that $\mu_{02(tb)}^x$ does not tend to zero as N increases, which clearly illustrates that it is incorrect to use a thermobattery with radiative heat transfer.

The equation analogous to (13) can be derived for radiative heat transfer with a uniform θ distribution for the couples, with the corresponding N dependence of μ_{02}^θ. Figure 1 shows this, with $\mu_{02(z)}^\theta$ corresponding to the flux density being integrated approximately on irradiation and substitution, while $\mu_{02(e)}^\theta$ corresponds to the integration in substitution being performed exactly by analytic means on the basis of uniformity in the cavity heating. One needs to choose a mode of integration having the same order of accuracy for both states, since otherwise the error of measurement can be increased because the residual terms in the quadrature formulas are not equivalent.

All the μ_{02} in Fig. 1 are given in modulus, although μ^x_{02} and μ_{02}^θ differ in sign for a given aperture angle. This is done to give a compact figure and also for convenience in comparing the absolute values.

The calculations confirm that the thermocouple disposition and the data processing substantially affect the main error associated with electrical replacement. One has to choose the couple positions and the quadrature formula for the numerical integration to minimize that error. If one lacks adequate information on the absorbed radiation distribution at the inner wall, one can use only general evaluations for the accuracy of the quadrature formulas.

Numerical-analysis theory shows that the highest integration accuracy for a fixed number of nodes N is provided for a fairly smooth function by a Gauss quadrature formula [6, 7], which has the highest algebraic order of accuracy (with N nodes, it is accurate for polynomials of degree 2N − 1). Here $U_1(1,\ x)$ and $U_{1p}(1,\ x)$ are smooth functions, as is evident from the forms given in [4, 8] and from the fact that they are solutions to an elliptic equation with smooth boundary values for ρ = 1 [7].

When a Gauss quadrature is used, the result for convective-radiative transfer is

$$\sum_{k=1}^{N} A_k V_1(P_1,\ x_k) = 2V_2(P_2), \tag{16}$$

in which $A_k = 2(1 - x_k^2)^{-1}\ [P_N'(x_k)]^{-2}$ are the weighting coefficients in the Gauss formula and x_k are the roots of the Legendre polynomial $P_N(x)$.

As A_k and x_k are uniquely determined by the number of nodes (thermocouples) N, the design for a given N should provide for independent measurement of the readings of each couple.

To realize (16) and the other equations, one can use a data-acquisition system such as the Aksamit, which has 48 input voltage channels (thermo-emf) and six analog output channels, one of which can be used in automatic substitution control.

The basic systematic error μ_{01}^G in implementing (16) is

$$\mu_{01}^{G} = -\frac{\varepsilon_1 \mathrm{Bi}}{2}\sum_{k=1}^{N} A_k \sum_{m=1}^{\infty} C_m(x_k). \tag{17}$$

For radiative transfer, the equivalence criterion with a Gauss formula is

$$\sum_{k=1}^{N} A_k\left[\frac{V_1(P_1,\ x_k)}{\beta} + T_0\right]^4 = 2\left[\frac{V_2(P_2)}{\beta} + T_0\right]^4. \tag{18}$$

When (18) is realized, μ_{02}^G is

$$\mu_{02}^{\mathrm{cr}} = 1 + \frac{2\pi R^2\varepsilon_2\sigma T_0^4}{P_1}\left[2 - \sum_{k=1}^{N} A_k U_{1p}^4(1,\ x_k)\right]. \tag{19}$$

Figure 1 also shows the N dependence of μ_{02}^G corresponding to (19). A Gauss quadrature enables one to provide a basic systematic error less than 0.1% even with a narrow radiation beam ($\theta_0 = 10°$) and with only 6-8 thermocouples. With the same N, $\mu_{02(a)}^{\theta}$ is larger by a factor 5-7 and μ_{02}^x by almost an order of magnitude.

A thermobattery with radiative transfer would increase $\mu_{02(Tb)}^x$ by a factor of 30-40 by comparison with μ_{02}^G for the same N (Fig. 1).

Calculations from (17) for linear transfer show that using (16) instead of (3) or (11) provides a substantial increase in the accuracy, particularly for small θ_0, i.e., for a laser beam or a highly collimated thermal radiation flux. To determine the number of couples needed to provide a given error κ as a function of θ_0, we used (9), (12), and (17) to determine the minimum N providing $|\mu_{01}| \le \kappa$. Figure 2 shows the θ_0 dependence of N for three values of κ with the thermocouples uniformly spaced in x (Δx), and in θ ($\Delta\theta$) or in accordance with the Gauss formula with a limiting power of 50 W for the given radiometer model.

Implementing (16) by means of a data-acquisition system instead of the (3) thermobattery with $\theta_0 = 10$-$30°$ enables one to reduce the number of couples for a given κ by a substantial factor. However, as θ_0 increases, the advantages from the Gauss quadrature diminish, and they persist for $\theta_0 = 100$-$120°$ only for $\kappa < 0.05\%$, which shows that for large θ_0, i.e., small tangential temperature gradients, the error in the numerical integration is largely independent of the integration method and a Gauss formula has advantages only at the level of the basic systematic error, which is comparable with the other errors in the substitution method. Therefore, for large θ_0, a Gauss quadrature can be replaced by other numerical integration methods.

However, for large θ_0, a Gauss quadrature has the advantage that the A_k are always positive and normalized such that their sum is 2 in the range [−1, 1]. Therefore, the additional integration error that could accumulate with sign-varying A_k due to inaccuracy in measuring the temperature of each couple in the present case for any N will not exceed twice the maximum error for one couple. Also, a Gauss quadrature leads to algorithms free from saturation, which is also important in a data-acquisition system.

If there are no data on E(x), the N that guarantees a given κ with a given mode of substitution may be derived from the same formulas for μ_{01} and μ_{02} by substituting for the temperature pattern in which E(x) is specified as a delta function, i.e., $E(x)\equiv\delta(1 - x)$.

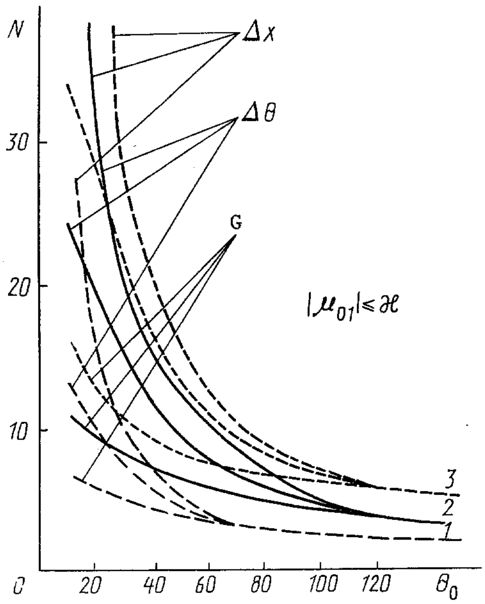

Fig. 2. The θ_0 dependence of N for $\kappa = 0.5$ (1), 0.1 (2), 0.05% (3) with the couples uniformly disposed in x (Δx) and in θ ($\Delta\theta$) and also in accordance with the Gauss formula g; θ_0 in degrees.

For $U_1(1, x)$ for example, replacing E(x) by a delta function is equivalent to $F_m \equiv 1$ in (6), which corresponds to the largest tangential temperature gradients for the given cavity parameters, so the N found from this condition and $|\mu_{01}| \leq \kappa$ will certainly be sufficient for any other form of E(x). One should proceed similarly in calculating $U_{1p}(1, x)$ by the [4] method.

This method of reducing the basic systematic error in substitution is applicable not only to a spherical cavity but also to other shapes, and this applies not only to replacing a thermobattery by a set of isolated thermocouples, whose readings are processed by an optimal numerical-integration algorithm, but also when one uses a Gauss quadrature, since smoothness in temperature patterns is a general feature in thermal conduction. With other forms of cavity, the integration range is naturally altered, but there is no mathematical difficulty [9] in converting A_k and x_k from the range [−1, 1] to any interval [b, a].

This approach can be supplemented with other ways of minimizing the basic substitution error, one of which has been described in [10].

Notation. R and R_1 outside and inside radii of sphere; d, input hole diameter; t(R, x) and T_0, temperatures of the outer surface of the sphere and environment; β, thermocouple sensitivity; ε_1 and ε_2, blacknesses of the inner and outer surfaces of the detecting element; λ_s, thermal conductivity of the outer layer in the spherical shell; V_1 and V_2, emfs of thermocouples on irradiation and substitution; θ_0, aperture angle of flux governing angular boundary of irradiation spot on cavity wall; $\rho = r/R$ and θ, radial and polar coordinates in spherical system; σ, Stefan's constant; $P_n(x)$, Legendre polynomials of the first kind and order n.

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TEMPERATURE MEASUREMENT IN A REDUCED-PRESSURE SUBSONIC OXYGEN JET

E. B. Georg and M. I. Yakushin

UDC 533.9.082.5

An induction plasmotron has been used to produce a stream of oxygen plasma, whose temperature distributions in the axial and radial directions have been measured at low pressure.

There are very few papers on oxygen plasmas, mainly because of difficulties in making them, as oxygen is a gas representing an explosion hazard, and when it is used at reduced pressures, one needs a mechanical pump filled with vacuum oil.

The first in which an oxygen plasma made in an induction discharge is mentioned goes back to 1961 [1]. All the measurements were made at atmospheric pressure in a plasma based on a mixture of argon with oxygen.

So far as we are aware, no experimental study has been made on induction plasmotrons with the use of oxygen plasmas at low pressures, although the topic is clearly important on account of research in gas-phase kinetics, catalysis, nonequilibrium heat transfer, and high-temperature materials science.

The present paper continues researches published in [2, 3] and relates to temperature measurements on free subsonic jets of air and nitrogen at reduced pressures produced with a VGU-2 induction plasmotron.

The VGU-2 has been used with a discharge channel 6 cm in diameter under the following conditions: anode power inputs 31.6 and 34.5 kW, oxygen flow rates 2.9-3.06 g/sec, pressure range $8 \cdot 10^3$-10^4 Pa, and flow speed at the axis near the end of the channel 90-110 m/sec.

The free jet was imaged on the slit of a DFS-452 spectrograph (dispersion 1.6 nm/mm) or that of a McPherson monochromator (dispersion 0.8 nm/mm) with photographic recording. The plasma parameters were measured along the axis and the radius near the end of the discharge channel.

The spectra were recorded at 200-800 nm. In the UV range, the spectrum is radiated in the main by the Schumann–Runge O_2 bands at 310-450 nm and the second negative system of O_2^+ at 260-310 nm, while in the visible and red regions, one gets the emission from atomic oxygen O I. Impurities were absent from the spectrum.

The plasma jet had a uniform blue emission (O_2) with 31.6 kW anode input (mode I). At 34.5 kW (mode II), the jet structure altered: a narrow core with bright emission appeared and the peripheral part acquired a yellow–green color, the extent of the core varying with the pressure, oxygen flow rate, and energy disposition. The spectral pattern was that in I, the main source was provided by oxygen molecules, with very weak emission from atomic oxygen, mainly in the two lines at 777 and 616 nm. As the power increased, the molecular spectrum weakened, and very strong lines from atomic oxygen appeared (excitation potentials over 12 eV), with the intensity there also substantially dependent on the pressure for a given power. As the pressure was reduced from 10^4 to $8 \cdot 10^3$ Pa, the intensity increased, while the lines vanished at $2 \cdot 10^4$ Pa, and the molecular emission strengthened.

A water-cooled copper holder containing the sensors or specimens was used in research on nonequilibrium heat transfer in the jet, so it was necessary to establish how the holder

Mechanics Problems Institute, USSR Academy of Sciences, Moscow. Translated from Inzhenerno-fizicheskii Zhurnal, No. 2, pp. 318-321, February, 1991. Original article submitted January 29, 1990.